Coupled Mode Analysis of a Finline

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Abstract —A theoretical analysis of a unilateral finline loaded with arbitrary inhomogeneous lossy dielectric material is presented. The rigorous coupled mode approach is used. The electromagnetic field in the line is expressed in terms of the modes of a ridged waveguide and the problem is transformed to a matrix eigenvalue equation. Approximate expressions are derived for investigating the properties of the fundamental mode in finlines loaded with dielectric slabs. Dispersion characteristics, the characteristic impedance, and the attenuation due to dielectric losses and the finite conductivity of metal coating are computed for various line configurations. The numerical results are compared with data obtained by means of the spectral-domain method, proving the validity and usefulness of the proposed approach.

I. Introduction

SINCE THE finline was first proposed in 1974 by Meier [1] as a new transmission line for millimeter-wave integrated circuits, E-plane printed waveguides have been established for use in low-cost millimeter systems. The application of finlines has been discussed by many authors and various theoretical analyses have been proposed to study their general properties. The existing literature concerning planar structures, finlines in particular, is vast and only a few of the publications will be referred to in this paper. More exhaustive lists of references can be found in the comprehensive reviews published by Solbach [2] and Jansen [3].

Approximate solutions for the finlines, including dispersion diagrams and the characteristic impedance of the fundamental mode, can be obtained using the equivalent model approach [4], curve fitting [5], the modified TLM method [6], or the modified transverse resonance method [7]. Exact methods stem from the full-wave, hybrid-mode formulation of the boundary value problem and allow one to obtain a field solution for dominant as well as higher order modes necessary for characterization of finline discontinuities. Among the various techniques, the spectral-domain method (SDM), first proposed by Itoh and Schmidt [8], proved to be particularly suitable for numerical implementation and since then has become a widely accepted standard. This technique was then developed and results of the investigation of various properties of finlines, including the characteristic impedance [9], attenuation due to dielectric and conductor losses [10], and the effect of finite metallization thickness [11], have been published. Among other rigorous approaches at least one, namely the mode-matching procedure [12], [13], should be mentioned. This method, although less numerically efficient then the SDM, can be used to investigate more general finline configurations with more than one dielectric region, finite fin thickness, and substrate mounting grooves [13].

Both exact and approximate methods are used, with certain exceptions [10], to study idealized lossless structures. Moreover, the homogeneity of the dielectric substrate in the direction parallel to the fins is a prerequisite to most of the theoretical analyses. Among the aforementioned procedures, only the mode-matching method is theoretically able to provide an accurate solution for finline structures loaded with dielectric slabs inhomogeneous in both the E and H planes.

In this paper we propose a simple and efficient method for analyzing a finline loaded with arbitrary inhomogeneous lossy dielectric material. The analysis is based on the coupled mode method for investigating nonideal waveguides [14], [17] or structures containing isotropic or anisotropic inserts [15], [16]. The electromagnetic field in the line is expanded into series of eigenfunctions of a ridged waveguide, and the boundary value problem is transformed into a matrix eigenvalue equation. The proposed method, in its full form, is accurate. Moreover, it also offers useful, almost analytical approximate expressions for investigating the properties of the fundamental mode in finlines loaded with dielectric slabs, including dispersion characteristics, the characteristic impedance, and the attenuation due to dielectric losses and the finite conductivity of metal coating.

II. Analysis

The cross section of the analyzed line is shown in Fig. 1(a). The solution to the boundary value problem describing electromagnetic wave propagation in the z direction in the structure under investigation can be found using the coupled mode method [14]–[16]. In this procedure the fields in the analyzed guide are expressed in terms of the fields of a second, basis waveguiding structure whose modal solutions are known. The analyzed structure may be regarded as a modification of a ridged guide with ridges of zero thickness (Fig. 1(b)). This affinity allows us to assume that the electromagnetic field in the investigated structure can be expressed in terms of the fields of ridged guide. Let $\vec{\mathscr{E}}_n$ and $\vec{\mathscr{H}}_l$ denote the eigenfunctions corresponding to the

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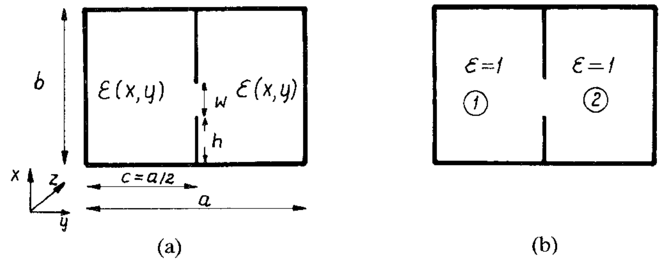

Fig. 1. Cross-sectional view of (a) the analyzed finline and (b) basis structure.

modes of the ridged guide. The eigenfunctions are normalized and orthogonal; i.e., they satisfy the following relation:

$$\int_\Omega \vec{a}_z \cdot \left(\vec{\mathscr{E}}_n \times \vec{\mathscr{H}}_i^* \right) d\Omega = \delta_{in} \tag{1}$$

where δ_{in} is the Kronecker delta.

According to the coupled mode procedure we expand the transverse field components in the analyzed guide into series of the eigenfunctions:

$$\vec{E}_t = \sum_n v_n(z) \vec{\mathscr{E}}_{tn}(x,y) \qquad \vec{H}_t = \sum_n i_n(z) \vec{\mathscr{H}}_{tn}(x,y) \tag{2}$$

where $v_n(z) = v_n e^{-j\beta_f z}$ and $i_n(z) = e_n e^{-j\beta_f z}$ are the unknown amplitudes and β_f is the propagation constant of the analyzed guide. Using (2) and bearing in mind the orthonormality relation (1), we transform Maxwell's equations to the coupled mode equations:

$$\frac{\partial}{\partial z} v_i = -j\beta_i Z_i i_i - j\sum_n i_n K_{zni}$$

$$\frac{\partial}{\partial z} i_i = -j\beta_i Y_i v_i - j\sum_n v_n K_{tni}, \qquad i = 1,2,\cdots. \tag{3}$$

In the above equations β_i, Y_i, and Z_i denote the propagation constant, the wave admittance, and the impedance of the ith mode of the ridged guide, respectively, whereas K_{zni} and K_{tni} are the coefficients describing the couplings between modes of the ridged guide that result from the perturbation caused by the dielectric insert. The coupling coefficients are given by the following equations:

$$K_{tni} = \frac{k_0}{\eta_0} \int_\Omega \left[(\epsilon - 1) \vec{\mathscr{E}}_{tn} \cdot \vec{\mathscr{E}}_{ti}^* \right] d\Omega$$

$$K_{zni} = \frac{k_0}{\eta_0} \int_\Omega \left[\frac{\epsilon - 1}{\epsilon} \mathscr{E}_{zn} \mathscr{E}_{zi}^* \right] d\Omega \tag{4}$$

where Ω is the cross section of the analyzed guide and k_0 and η_0 are, respectively, the wavenumber and the intrinsic impedance of free space.

Equations (3) can be cast into matrix form, yielding

$$\left[\underline{\underline{Q}} - \beta_f \underline{\underline{1}} \right] \begin{bmatrix} \underline{v} \\ \underline{i} \end{bmatrix} = 0 \tag{5}$$

with

$$\underline{\underline{Q}} = \begin{bmatrix} 0 & \underline{\underline{Q}}_{12} \\ \underline{\underline{Q}}_{21} & 0 \end{bmatrix}$$

and

$$Q_{12}^{ni} = \beta_i Z_i \delta_{ni} + K_{zni} \qquad Q_{21}^{ni} = \beta_i Y_i \delta_{ni} + K_{tni}$$

where $\underline{\underline{1}}$ denotes the unity matrix.

Equation (5) constitutes a matrix eigenvalue problem, which implies that the unknown propagation constant β_f as well as the expansion coefficients v_i, i_i can be easily found as the eigenvalues and eigenvectors of matrix $\underline{\underline{Q}}$.

A. Eigenfunctions of a Ridged Guide

In order to solve (5) we have to determine the eigenfunctions of the basis structure. A ridged waveguide supports TE and TM modes. The eigenfunctions corresponding to these modes can be derived from the scalar potentials ψ and ϕ. Since there are no closed-form expressions for the scalar potentials in the ridged guide, we use for their determination the spectral-domain procedure [8]–[11], [19]. Expanding into Fourier series the unknown potentials in regions 1 and 2 on both sides of the ridges and taking into account the boundary conditions on the screening walls as well as the symmetry properties with respect to the plane $y = c$, we obtain

$$\psi_1 = \sum_{n=0} C_n^h \cosh(\alpha_n^h y) \cos(k_n x)$$

$$\psi_2 = -\sum_{n=0} C_n^h \cosh\left[\alpha_n^h (a - y)\right] \cos(k_n x) \tag{6}$$

and

$$\phi_1 = \sum_{n=1} C_n^e \sinh(\alpha_n^e y) \sin(k_n x)$$

$$\phi_2 = \sum_{n=1} C_n^e \sinh\left[\alpha_n^e (a - y)\right] \sin(k_n x) \tag{7}$$

where

$$\alpha_n^{h(e)} = \sqrt{k_n^2 - p_{h(e)}^2} \qquad p_{h(e)}^2 = k_0^2 - \beta_{h(e)}^2$$

with $k_n = n\pi/b$. In the above expressions the symbols h and e correspond to TE and TM modes, respectively.

The eigenfunctions $\vec{\mathscr{E}}_n$ and $\vec{\mathscr{H}}_i$ can be obtained from the following relations [17]:

$$\vec{\mathscr{H}}_t^h = -\nabla_t \psi \qquad \vec{H}_z^h = -j\frac{p_h^2}{k_0 \eta_0} \psi \vec{a}_z \qquad \vec{\mathscr{E}}_t^h = \vec{\mathscr{H}}_t^h \times \vec{a}_z \tag{8}$$

$$\vec{\mathscr{E}}_t^e = -\nabla_t \phi \qquad \vec{\mathscr{E}}_z^e = -j\frac{p_e^2 \eta_0}{k_0} \phi \vec{a}_z \qquad \vec{\mathscr{H}}_t^e o = -\vec{\mathscr{E}}_t^e \times \vec{a}_z \tag{9}$$

where $\nabla_t = (\partial/\partial x)\vec{a}_x + (\partial/\partial y)\vec{a}_y$.

The continuity conditions in the plane $y = c$ bring about a functional equation relating current densities on the ridges to the electric field in the slot. The functional equation is solved using the Galerkin procedure in the Fourier domain. This technique requires the expansion of the field inside the slot into series of known basis functions with unknown coefficients. As a result of the procedure we obtain an infinite set of homogeneous equations:

$$\sum_{m=1} a_m^h \sum_{n=0} (2-\delta_{n0})\tilde{e}_{x_{mn}}\tilde{e}^*_{x_{jn}} \frac{\coth(\alpha_n^h c)}{\alpha_n^h} = 0, \qquad j=1,2,\cdots \tag{10}$$

for TE modes and

$$\sum_{m=1} a_m^e \sum_{n=1} \tilde{e}_{x_{mn}}\tilde{e}^*_{x_{jn}} \frac{\alpha_n^e \coth(\alpha_n^e c)}{k_n^2} = 0, \qquad j=1,2,\cdots \tag{11}$$

for TM modes. In formulas (10) and (11) $\tilde{e}_{x_{mn}}$ denotes the nth Fourier term of the mth basis function and a_m^h and a_m^e are the unknown amplitudes. The determinant of the above set of equations is set to zero, yielding the dispersion equation for the TE or TM modes of the ridged guide. Once the root of the dispersion equation β is found, it is substituted into (10) and (11) and the resulting set of equations is solved, giving the amplitudes of basis functions.

The yet unknown expansion coefficients C_n^h and C_n^e are related to the Fourier transforms of the field between fins by the following equations:

$$C_n^h = \frac{2-\delta_{n0}}{b} \frac{\sum_{m=1} a_m^h \tilde{e}_{x_{mn}}}{\alpha_n^h \sinh(\alpha_n^h c)} = a_1^h \tilde{C}_n^h \tag{12}$$

$$C_n^e = \frac{2}{b} \frac{\sum_{m=1} a_m^e \tilde{e}_{x_{mn}}}{k_n \sinh(\alpha_n^e c)} = a_1^e \tilde{C}_n^e. \tag{13}$$

In order to ensure the orthonormality of the eigenfunctions we have to normalize amplitudes a_1^h and a_1^e. For this purpose we use the following normalization conditions for the scalar potentials [17]:

$$\int_\Omega \psi\psi^* \, d\Omega = \frac{1}{P_h^2} \qquad \int_\Omega \phi\phi^* \, d\Omega = \frac{1}{P_e^2}. \tag{14}$$

If these conditions are fulfilled the eigenfunctions are also normalized. Using (12) and (13) together with (6), (7), and (14), we obtain

$$a_1^h = \left[\frac{ab}{2} p_h^2 \sum_{n=0} \frac{|\tilde{C}_n^h|^2}{2-\delta_{n0}} \left(\frac{\sinh(2\alpha_n^h c)}{2\alpha_n^h} + 1\right)\right]^{(-1/2)} \tag{15}$$

$$a_1^e = \left[\frac{ab}{2} p_e^2 \sum_{n=1} \frac{|\tilde{C}_n^e|^2}{2} \left(\frac{\sinh(2\alpha_n^e c)}{2\alpha_n^e} - 1\right)\right]^{(-1/2)}. \tag{16}$$

B. Single-Mode Approximation

Using the procedure outlined above, we may obtain a rigorous solution for a wide range of waveguiding structures. The accuracy of the method depends on the number of eigenfunctions taken into account in the field expansion. Note that up to now the analysis was general. In particular we did not assume any particular shape of the dielectric insert. Presently we shall concentrate on the analysis of lines loaded with slabs of rectangular cross section and symmetrical with respect to the plane $x = b/2$ (Fig. 2). Finlines operating in the millimeter-wave range use thin, low-permittivity dielectric substrate. Thus, we may assume that the dielectric perturbs mainly the fundamental TE_1 mode of the ridged guide. Owing to the symmetry of the structure we may also put $k_n = 2n\pi/b$. The coupled mode equations take, in this case, the following form:

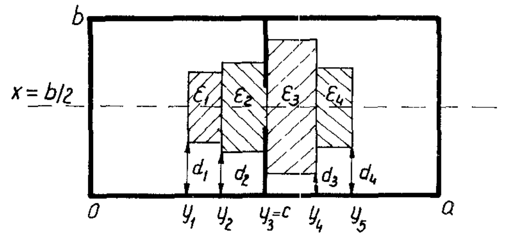

Fig. 2. Cross-sectional view of a symmetrical structure of a finline loaded with dielectric slabs.

$$\beta_f v_1^h = \beta_1^h Z_1^h i_1^h$$
$$\beta_f i_1^h = (\beta_1^h Y_1^h + K_{t11}) v_1^h \tag{17}$$

with $Z_1^h = k_0\eta_0/\beta_1^h$. Note, that since $E_z = 0$, K_{z11} equals zero.

Additionally, in this paper we shall use only a single function to approximate the electric field between fins. The determinantal dispersion relation then becomes a transcendental equation which can be readily solved even on a personal computer.

C. Propagation Constant and Characteristic Impedance

Having determined the eigenfunctions of the basis guide, we can compute the coupling coefficient K_{t11}. Let us assume that the investigated line is loaded with L slabs. In this case we have

$$K_{t11} = \frac{k_0}{\eta_0} \sum_{\iota=1}^{L} (\epsilon_\iota - 1) F_\iota \tag{18}$$

with

$$F_\iota = \int_{d_\iota}^{b-d_\iota} \int_{y_\iota}^{y_{\iota+1}} \vec{\mathscr{E}}_{t1} \cdot \vec{\mathscr{E}}_{t1}^* \, dx\, dy. \tag{19}$$

Using (8) together with (6) we obtain

$$F_\iota = \sum_{n=1} \sum_{m=1} k_n k_m C_n^h C_m^h I^{cc}_{y_{nm}} I^{ss}_{x_{nm}} + \sum_{n=0} \sum_{m=0} \alpha_n^h \alpha_m^h C_n^h C_m^h I^{ss}_{y_{nm}} I^{cc}_{x_{nm}} \tag{20}$$

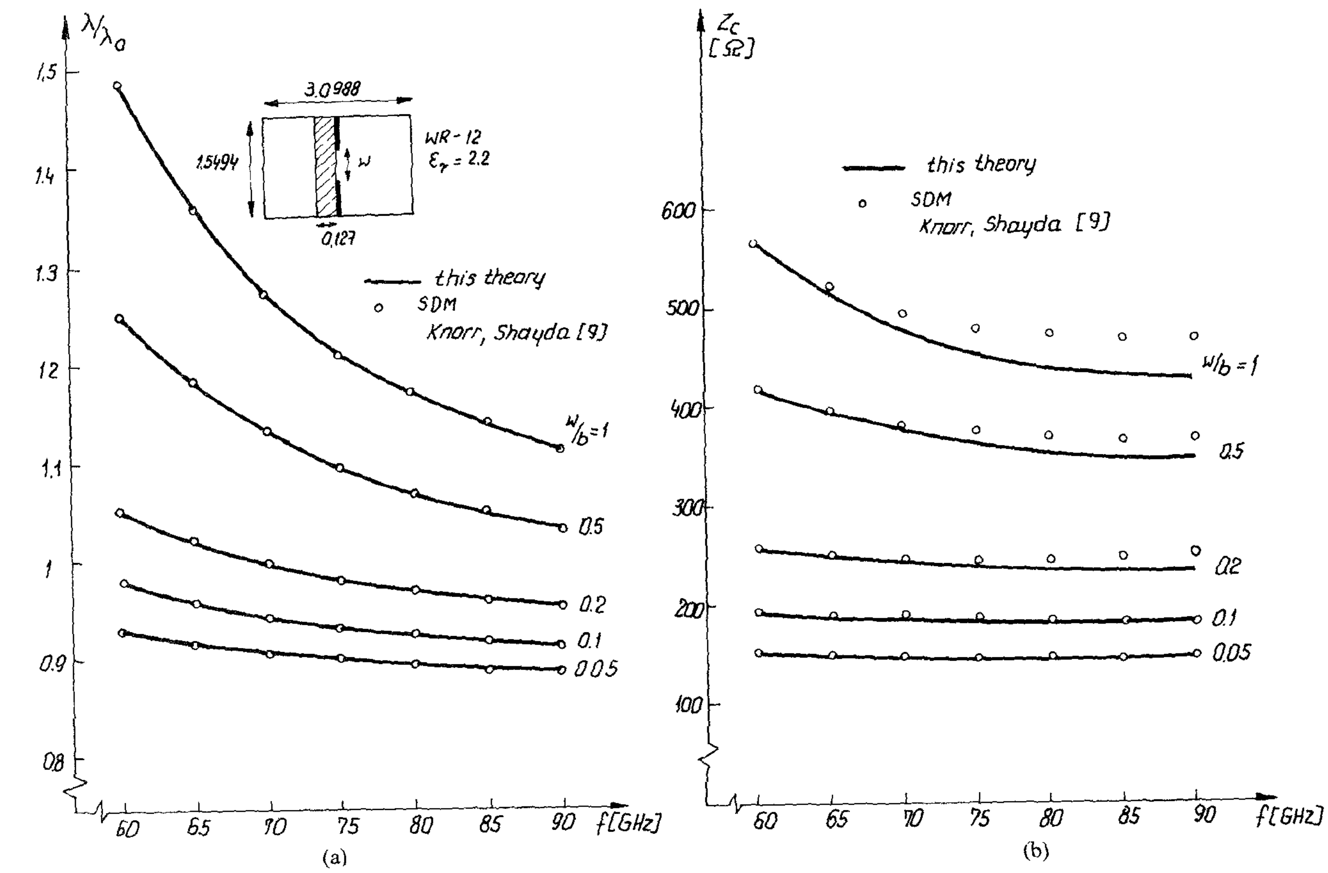

Fig. 3. Comparison of this method with SDM [9] ($\epsilon = 2.2$). (a) Wavelength ratio λ/λ_0. (b) Characteristic impedance. (Dimensions in mm, fins and slot are centered.) (a) (b)

where

$$I^{cc}_{x_{nm}} = \int_{d_i}^{b-d_i} \cos(k_n x)\cos(k_m x)\, dx$$

$$I^{ss}_{x_{nm}} = \int_{d_i}^{b-d_i} \sin(k_n x)\sin(k_m x)\, dx \tag{21}$$

$$I^{cc}_{y_{nm}} = \int_{l_1}^{l_2} \cosh(\alpha^h_n y)\cosh(\alpha^h_m y)\, dy$$

$$I^{ss}_{y_{nm}} = \int_{l_1}^{l_2} \sinh(\alpha^h_n y)\sinh(\alpha^h_m y)\, dy \tag{22}$$

with $l_1 = y_i, l_2 = y_{i+1}$ if $y_{i+1} \leqslant c$ and $l_1 = c - y_{i+1}$, $l_2 = c - y_i$ if $y_i > c$.

Once the coefficient K_{n1} is found we can obtain from (17) the following approximate analytic expression for the propagation constant in the investigated line:

$$\beta_f^2 = \beta_1^{h^2} + k_0\eta_0 K_{n1}. \tag{23}$$

The characteristic impedance is defined as

$$Z_c = \frac{|U|^2}{P} \tag{24}$$

where

$$U = \int_{(b-w)/2}^{(b+w)/2} E_{x|_{y=c}}\, dx = v_1^h \int_{(b-w)/2}^{(b+w)/2} \mathscr{E}_{x|_{y=c}}\, dx = v_1^h a_1^h \tilde{e}_{x_{10}}$$

$$P = \int_\Omega \vec{a}_z \cdot (\vec{E} \times \vec{H}^*)\, d\Omega$$

$$= v_1^h i_1^{*h} \int_\Omega \vec{a}_z \cdot (\vec{\mathscr{E}} \times \vec{\mathscr{H}}^*)\, d\Omega = v_1^h i_1^{*h}.$$

Substituting U and P into (24) we obtain

$$Z_c = Z_f^h |a_1^h|^2 |\tilde{e}_{x_{10}}|^2 \tag{25}$$

with $Z_f^h = k_0\eta_0/\beta_f$.

D. Conductivity and Dielectric Losses

Attenuation due to the finite conductivity of the fins and screening walls can be found by means of the perturbation method [17]. The conductivity losses may be expressed as

$$\alpha_e = \frac{R_m \oint_{\partial\Omega} |\vec{n} \times \vec{H}|^2\, dl}{2P} \tag{26}$$

where $R_m = \sqrt{k_0\eta_0/2\sigma}$. In the above formulas σ is the conductivity of the metal and $\vec{n}$ is a unit vector normal to

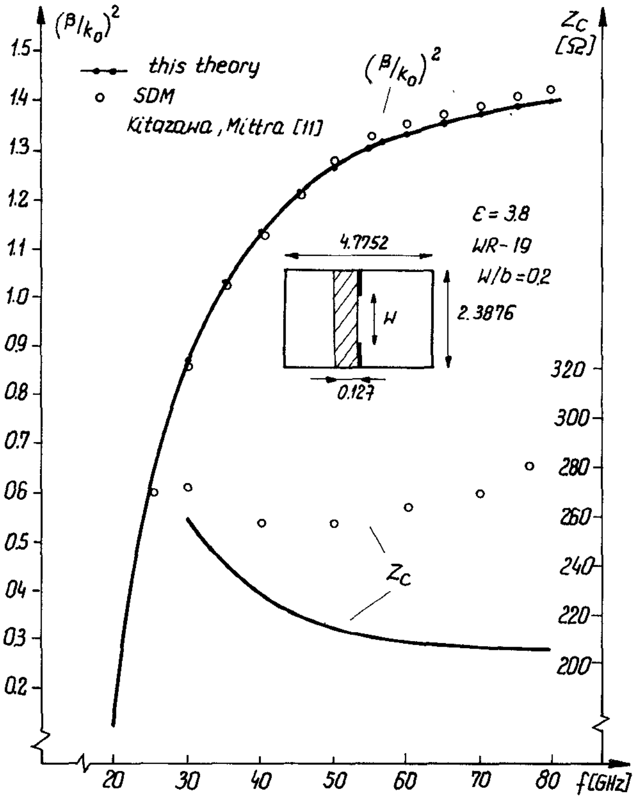

Fig. 4. Comparison of this method with SDM [11] ($\epsilon = 3.8$. Dimensions in mm, fins and slots are centered.)

the contour $\partial\Omega$. Substituting (8) and (26) we obtain

$$\alpha_c = \frac{R_m}{2k_0\eta_0\beta_f}\left(\beta_f^2\oint_{\partial\Omega}|\nabla_t\psi|^2\,dl + p_h^4\oint_{\partial\Omega}|\psi|^2\,dl\right). \quad (27)$$

Detailed expressions for the integrals appearing in formula (27) are given in the Appendix.

Dielectric losses can be obtained directly from (23). Assuming that the relative permittivity of the ith slab is given by

$$\hat{\epsilon}_i = \epsilon_i - j\epsilon_i \tan\delta_{\epsilon i}$$

with $\tan\delta_{\epsilon i}$ being the loss tangent, and substituting it into (23) we get

$$\gamma_f^2 = \beta_1^{h^2} + k_0^2 \sum_{i=1}^{L} (\epsilon_i - j\epsilon_i \tan\delta_{\epsilon i}) F_i.$$

Thus, the dielectric losses are given by

$$\alpha_d = \frac{k_0^2 \sum_{i=1}^{L} \epsilon_i \tan\delta_{\epsilon i} F_i}{2\beta_f}. \quad (28)$$

III. Numerical Results

The analysis presented in the previous section was implemented on a personal computer. To verify the accuracy of the method, the numerical results were compared with data available from the literature. Most of the curves presented in this paper were obtained using the following functions approximating the electric field between fins:

$$e_{x1} = 1 \quad (29)$$

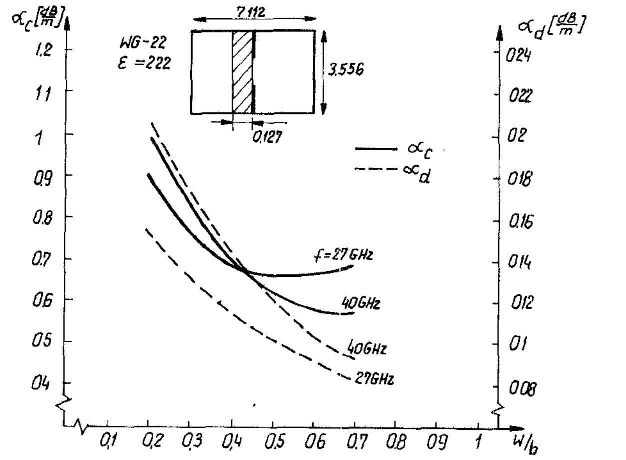

Fig. 5. Conductivity loss α_c and dielectric loss α_d versus w/b. ($\tan\delta_\epsilon = 2.0\cdot10^{-4}$, $\sigma = 3.33\cdot10^4$ 1/Ω mm. Dimensions in mm, fins and slot are centered.)

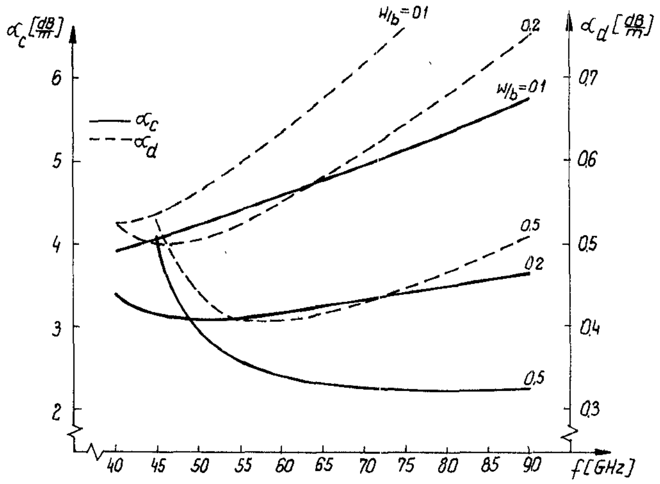

Fig. 6. Conductivity loss α_c and dielectric loss α_d versus frequency for the finline described in Fig. 3. ($\tan\delta_\epsilon = 2.0\cdot10^{-4}$, $\sigma = 3.33\cdot10^4$ 1/Ω mm. Dimensions in mm, fins and slot are centered.)

if $\omega/b > 0.8$ or

$$e_{x1} = \frac{1}{\sqrt{w^2 - (x - b/2)^2}} \quad (30)$$

if $\omega/b \leqslant 0.8$. The numerical results obtained by this method for a finline with a low-permittivity substrate ($\epsilon = 2.2$) are shown in Fig. 3 together with data published by Knorr and Shayda [9], who used the SDM. Very good agreement was observed for the propagation constant, and, provided that the slot is narrow compared to the line height, also for the characteristic impedance. The accuracy of the approximate formulas derived in this paper decreases with the frequency. This can be readily explained from the fact that the E_z field component, which becomes significant at higher frequencies, was ignored in the TE_1 mode approximation. Fig. 4 shows the influence of the permittivity of the substrate ($\epsilon = 3.8$) on the accuracy of the numerical results. The dispersion characteristic is accurate over a wide frequency range (2 percent difference at

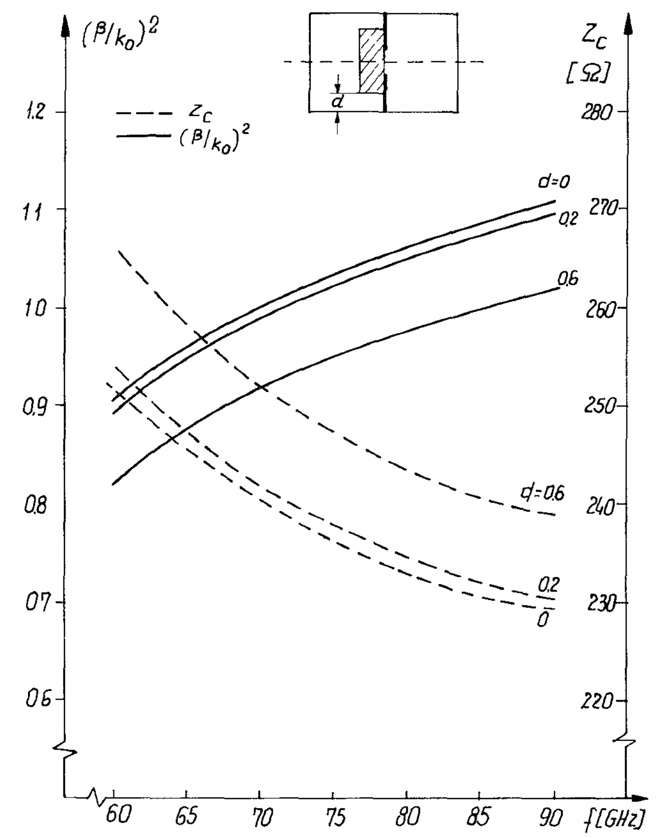

Fig. 7. Propagation characteristics of a finline with WR-12 shield (cf. Fig. 3) loaded with dielectric slabs ($\epsilon = 2.2$, $w/b = 0.2$, $t = 0.127$ mm. Dimensions in mm, fins and slot are centered.)

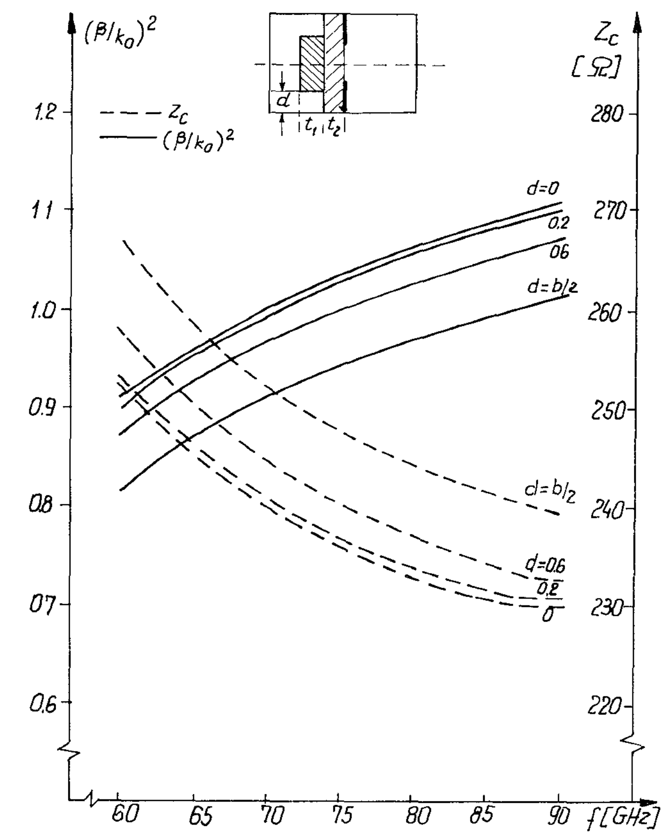

Fig. 8. Propagation characteristics of a finline with WR-12 shield (cf Fig. 3) loaded with dielectric slabs ($\epsilon_1 = \epsilon_2 = 2.2$, $w/b = 0.2$, $t_1 = t_2 = 0.0635$ mm. Dimensions in mm, fins and slot are centered.)

$f = 80$ GHz) in relation to the curve published by Kitazawa and Mittra [11]. However, as could have been expected, a greater discrepancy compared with the low-permittivity case is observed for the characteristic impedance curve.

The dielectric and conductivity losses for two different line configurations are displayed in Figs. 5 and 6. In order to avoid the nonconvergent series in conductivity loss computations, resulting from the unbounded magnetic field component near the edge of an infinitely thin conductor which leads to a non-square integrable integrand in perturbation formula (27), the function approximating the field between slots had to be modified. The following representation was taken in place of (30):

$$e_{x1} = \frac{1}{\left[w^2 - (x - b/2)^2\right]^{(1/3)}}. \tag{31}$$

The losses shown in Fig. 5 have values and character similar to the results published by Mirshekar-Syahkal and Davies [10]. Fig. 6 illustrates the frequency behavior of the losses for different slot widths. The attenuation due to the finite conductivity of the metal is approximately ten times higher than the dielectric loss. Both conductivity and dielectric losses increase as the slot width decreases.

Figs. 7–9 show the curves of the propagation constant and the characteristic impedance for structures loaded with dielectric material inhomogeneous in both the E plane and the H plane. The dielectric slab placed in the slot region reduces the impedance of the line. The stronger effect is observed for the slab placed directly over the slot (Fig. 9).

IV. Conclusions

A theoretical coupled mode analysis of a finline loaded with arbitrary inhomogeneous lossy dielectric material was presented. The method is exact if an infinite number of modes of a ridged waveguide are used to construct the field inside the line. Approximate, almost analytical expressions were derived for investigating the properties of the fundamental mode in finlines loaded with dielectric slabs. Propagation characteristics including dispersion diagrams, the characteristic impedance, and dielectric and conductivity losses were computed for various line configurations. The calculations were compared with results of spectral-domain analysis. Good agreement was observed for lines with low-permittivity substrates, proving the validity and usefulness of the proposed approach.

The method can be applied to study other finline configurations for which the SDM is inadequate. In particular, it allows one to investigate the propagation in finlines containing gyromagnetic material [18].

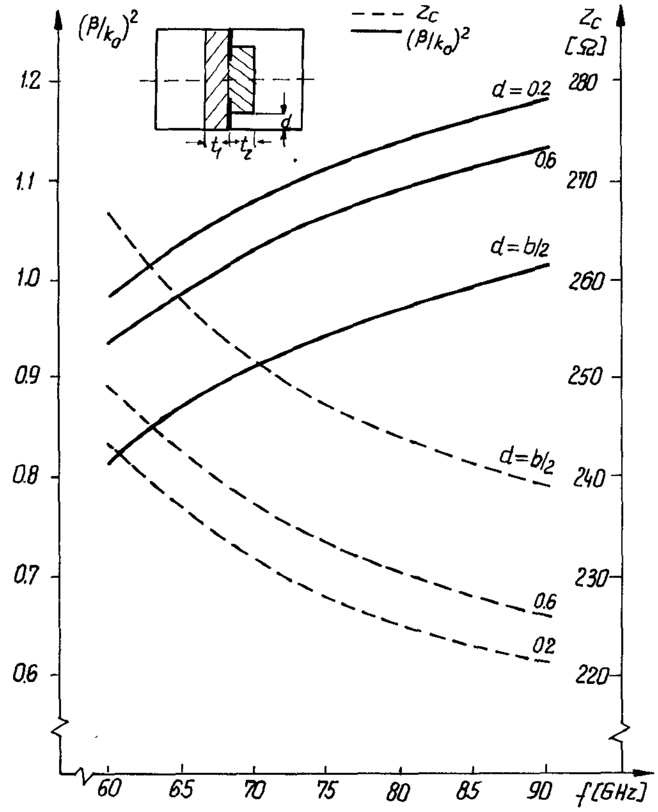

Fig. 9. Propagation characteristics of a finline with WR-12 shield (cf. Fig. 3) loaded with dielectric slabs ($\epsilon_1 = \epsilon_2 = 2.2$, $w/b = 0.2$, $t_1 = t_2 = 0.0635$ mm. Dimensions in mm, fins and slot are centered.)

Appendix

The contour integrals appearing in formula (27) are given by

$$\oint_{\partial\Omega} |\nabla_t \psi|^2 \, dl = 2(I_1 + 2I_2 + 2I_3) \tag{A1}$$

$$\oint_{\partial\Omega} |\psi|^2 \, dl = 2(I_4 + 2I_5 + 2I_6) \tag{A2}$$

where

$$I_1 = \sum_{n=1} C_n^{h^2} k_n^2 \int_0^b \sin^2(k_n x) \, dx \tag{A3}$$

$$I_2 = \sum_{n=0} \sum_{m=0} \alpha_n^h \alpha_m^h C_n^h C_m^h \int_0^c \sinh(\alpha_n^h y) \sinh(\alpha_m^h y) \, dy \tag{A4}$$

$$I_3 = \sum_{n=1} \sum_{m=1} k_n k_m C_n^h C_m^h \cosh(\alpha_n^h c) \cosh(\alpha_m^h c) \cdot \int_0^{(b-w)/2} \sin(k_n x) \sin(k_m x) \, dx \tag{A5}$$

$$I_4 = \sum_{n=0} C_n^{h^2} \int_0^b \cos^2(k_n x) \, dx \tag{A6}$$

$$I_5 = \sum_{n=0} \sum_{m=0} C_n^h C_m^h \int_0^e \cosh(\alpha_n^h y) \cosh(\alpha_m^h y) \, dy \tag{A7}$$

$$I_6 = \sum_{n=0} \sum_{m=0} C_n^h C_m^h \cosh(\alpha_n^h c) \cosh(\alpha_m^h c) \cdot \int_0^{(b-w)/2} \cos(k_n x) \cos(k_m x) \, dx \tag{A8}$$

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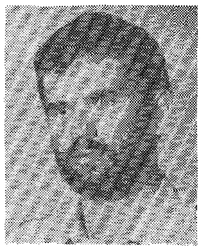

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